1 $y' = \frac{y}{x}$ $\int \frac{1}{y} dy = \int \frac{1}{x} dx$ $\ln|y| = \ln|x| + \tilde{c}$

$|y| = e^{\ln|x| + \tilde{c}} = |x| \cdot e^{\tilde{c}}$ $y_H = C \cdot x$

$y' = c'x + c \cdot 1$ in DGL $c'x + c = \frac{cx}{x} + x^2 \cos x$

$c' = x \cos x$ $c = \cos x + x \sin x + c$

$y = (\cos x + x \sin x + c) \cdot x$

$y(\pi) = 0 = (1 + c) \cdot \pi$ $\Rightarrow c = -1$

$y_S(x) = (\cos x + x \sin x - 1) \cdot x$

2 $\begin{pmatrix} 9 & -6 & 2 & -7 & 10 \\ 6 & -12 & 4 & -2 & 20 \\ 27 & -30 & 10 & -17 & 50 \\ -5 & 6 & -2 & 3 & -10 \end{pmatrix} \xrightarrow[\substack{-3\mathrm{I} \\ 9\mathrm{IV}+5\mathrm{I}}]{3\cdot\mathrm{II}-2\mathrm{I}} \begin{pmatrix} 9 & -6 & 2 & -7 & 10 \\ 0 & -24 & 8 & 8 & 40 \\ 0 & -12 & 4 & 4 & 20 \\ 0 & 24 & -8 & -8 & -40 \end{pmatrix} \rightarrow \begin{pmatrix} 9 & -6 & 2 & -7 & 10 \\ 0 & -3 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

μ λ

$x_4 = \lambda$, $x_3 = \mu$ $-3x_2 + \mu + \lambda = 5$

$x_2 = -\frac{5}{3} + \frac{1}{3}\mu + \frac{1}{3}\lambda$

$9x_1 - 6\left(-\frac{5}{3} + \frac{1}{3}\mu + \frac{1}{3}\lambda\right) + 2\mu - 7\lambda = 10$

$x_1 = \lambda$

$\vec{x} = \begin{pmatrix} 0 \\ -5/3 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1/3 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1/3 \\ 1 \\ 0 \end{pmatrix}$

z.B. $\lambda = 2, \mu = 0$ $\vec{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 3 \\ 0 \end{pmatrix}$

3 $\begin{vmatrix} -2-\lambda & 0 & -1 \\ 8 & -5-\lambda & -4 \\ -2 & 1 & -3-\lambda \end{vmatrix} \begin{matrix} -2-\lambda & 0 \\ 8 & -5-\lambda \\ -2 & 1 \end{matrix}$

$= (-2-\lambda)(-5-\lambda)(-3-\lambda) + 0 - 8 - 2(-5-\lambda) + 4(-2-\lambda) - 0$

$= -\lambda^3 - 10\lambda^2 - 33\lambda - 36$

$\lambda_1 = -3 \qquad (-\lambda^3 - 10\lambda^2 - 33\lambda - 36) : (\lambda + 3) = -\lambda^2 - 7\lambda - 12$

$\lambda_2 = -3 \quad \lambda_3 = -4$

EV zu $\lambda_1 = -3$: $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 8 & -2 & -4 & 0 \\ -2 & 1 & 0 & 0 \end{array}\right) \rightarrow v = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$-2 \cdot 1 + 1 v_2 = 0 \Rightarrow v_2 = 2$

ass V zu $\lambda_1 = -3$ $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 8 & -2 & -4 & 2 \\ -2 & 1 & 0 & 1 \end{array}\right) \rightarrow w = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$

$-2 \cdot 1 + v_2 = 1 \qquad v_2 = 3$

EV zu $\lambda_3 = -4$ $\left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 8 & -1 & -4 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right) \rightarrow v = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$-2 \cdot 1 + v_2 + 2 = 0 \qquad v_2 = 0$

$S = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix} \qquad S^{-1}AS = \begin{pmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

4 $f_x(x,y) = 3y - 4x + 1 = 0$

$f_y(x,y) = 3x - 2y = 0 \Rightarrow y = \frac{3}{2}x$

$3 \cdot \frac{3}{2}x - 4x + 1 = 0 \Rightarrow \frac{1}{2}x = -1 \quad x = -2$

$y = \frac{3}{2}(-2) = -3 \qquad z = 3 \cdot (-2)(-3) - 2(-2)^2 - (-3)^2 + (-2) + 5$

$z = 4$

$P(-2 | -3 | 4)$

$Hess_f(x,y) = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix} \qquad \begin{vmatrix} -4 & 3 \\ 3 & -2 \end{vmatrix} = 8 - 9 = -1 < 0$

$\Rightarrow$ P Sattelpunkt, keine rel. Extrema

5 Zylinderkoordinaten $x = r\cos\varphi \quad y = r\sin\varphi \quad z = z$

$dV = r\,dz\,dr\,d\varphi$

$$z = 8 - 2x^2 - 2y^2 = 8 - 2(r^2\cos^2\varphi) - 2r^2\sin^2\varphi$$

$$= 8 - 2r^2(\cos^2\varphi + \sin^2\varphi) = 8 - 2r^2 > 0 \Rightarrow r < 2$$

Nenner: $$\int_0^{2\pi}\int_0^2\int_0^{8-2r^2} r\,dz\,dr\,d\varphi = \int_0^{2\pi} d\varphi \cdot \int_0^2 r\cdot[z]_{z=0}^{8-2r^2}\,dr$$

$$= [\varphi]_0^{2\pi} \cdot \int_0^2 (8r - 2r^3)\,dr = [\varphi]_0^{2\pi} \cdot \left[8\frac{r^2}{2} - 2\frac{r^4}{4}\right]_0^2 = 16\pi$$

Zähler: $$\int_0^{2\pi}\int_0^2\int_0^{8-2r^2} z\cdot r\,dz\,dr\,d\varphi = \int_0^{2\pi} d\varphi \cdot \int_0^2 r\cdot\left[\frac{z^2}{2}\right]_{z=0}^{8-2r^2}\,dr$$

$$= 2\pi \cdot \int_0^2 (32r - 16r^3 + 2r^5)\,dr = 2\pi \cdot \left[32\frac{r^2}{2} - 16\frac{r^4}{4} + 2\frac{r^6}{6}\right]_0^2$$

$$= \frac{128}{3}\pi$$

$$z_s = \frac{\frac{128}{3}\pi}{16\pi} = \frac{8}{3} \Rightarrow S\left(0, 0, \frac{8}{3}\right)$$