1 $y' = \frac{y}{x}$ $\quad \int \frac{dy}{y} = \int \frac{dx}{x}$ $\quad \ln|y| = \ln|x| + c$ $\quad y = C \cdot x$

$y' = C'x + C \cdot 1 \underset{\uparrow \text{DGL}}{=} \frac{Cx}{x} + x\sin x \iff C' = \sin x \quad C = C - \cos x$

$y = (C - \cos x) \cdot x \qquad 1 = (C - \overbrace{\cos\tfrac{\pi}{2}}^{0}) \cdot \frac{\pi}{2} \iff \frac{2}{\pi} = C$

$y_s = x \cdot \left(\frac{2}{\pi} - \cos x\right)$

2 $y' - 2y = 0 \qquad y = C \cdot e^{2x}$

$y_p = Ae^{-3x} + Bx + C \qquad y_p' = -3Ae^{-3x} + B$

DGL: $-3Ae^{-3x} + B - 2(Ae^{-3x} + Bx + C) = 20e^{-3x} - 4x - 8$

$-5A = 20 \Rightarrow A = -4$

$2Bx = -4x \Rightarrow B = -2$

$B - 2C = -8 \Rightarrow C = \frac{-8-B}{-2} = \frac{-8+2}{-2} = 3$

$y = -4e^{-3x} - 2x + 3 + Ce^{2x}$

$0 = -4 + 3 + C \Rightarrow C = 1 \qquad y_s(x) = -4e^{-3x} - 2x + 3 + e^{2x}$

3 $\begin{vmatrix} 3-\lambda & -2 & 0 \\ 1 & 1-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{vmatrix} = -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0 \quad \lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$

EV zu $\lambda_1 = 1$ $\quad \begin{matrix} 2 & -2 & 0 \\ 1 & 0 & 1 \\ -1 & 2 & 1 \end{matrix} \qquad v = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

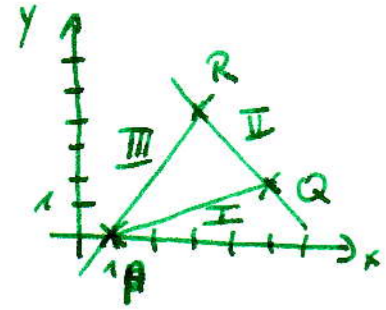
EV zu $\lambda_2 = 2$ $\quad \begin{matrix} 1 & -2 & 0 \\ 1 & -1 & 1 \\ \cdot & \cdot & \cdot \end{matrix} \qquad v = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

EV zu $\lambda_3 = 3$ $\quad \begin{matrix} 0 & -2 & 0 \\ 1 & -2 & 1 \\ \cdot & \cdot & \cdot \end{matrix} \qquad v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$S = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & -1 \end{pmatrix}$

$S^{-1}AS = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

4



I $\quad y = \frac{1}{2}x - \frac{1}{2}$

II $\quad y = -x + 7$

III $\quad y = 2x - 2$

$\int_1^3 \int_{\frac{1}{2}x-\frac{1}{2}}^{2x-2} f(x,y)\,dy\,dx + \int_3^5 \int_{\frac{1}{2}x-\frac{1}{2}}^{-x+7} f(x,y)\,dy\,dx$

5 $x = r\cos\varphi,\ y = r\sin\varphi,\ z = z$

$dV = r\,dz\,dr\,d\varphi$

$z = 4 - 4r^2\cos^2\varphi - 4r^2\sin^2\varphi = 4 - 4r^2(\cos^2\varphi + \sin^2\varphi) = 4 - 4r^2$

$x^2 + y^2 = r^2\cos^2\varphi + r^2\sin^2\varphi = r^2(\cos^2\varphi + \sin^2\varphi = r^2$

$$J_z = \int_0^{2\pi}\int_0^1\int_0^{4-4r^2} r^2 \cdot r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 \left[r^3 z\right]_{z=0}^{4-4r^2} dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^1 (4r^3 - 4r^5)\,dr\,d\varphi = \int_0^{2\pi}\left[4\frac{r^4}{4} - 4\frac{r^6}{6}\right]_{r=0}^1 d\varphi$$

$$= \int_0^{2\pi}\frac{1}{3}\,d\varphi = \left[\frac{1}{3}\varphi\right]_{\varphi=0}^{2\pi} = \frac{2}{3}\pi$$

6

$$\left(\begin{array}{cccc|c} 1 & 3 & 4 & 5 & 2 \\ 0 & -1 & -2 & -4 & 1 \\ 0 & -14 & -22 & -23 & -4 \\ 0 & -5 & -8 & -9 & -1 \end{array}\right) \qquad \left(\begin{array}{cccc|c} 1 & 3 & 4 & 5 & 2 \\ 0 & 1 & 2 & 4 & -1 \\ 0 & 0 & 6 & 33 & -18 \\ 0 & 0 & 2 & 11 & -6 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 4 & 5 & 2 \\ 0 & 1 & 2 & 4 & -1 \\ 0 & 0 & 1 & \frac{11}{2} & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_4 = \lambda$

$x_3 + \frac{11}{2}\lambda = -3 \qquad x_3 = -3 - \frac{11}{2}\lambda$

$x_2 + 2(-3 - \frac{11}{2}\lambda) + 4\lambda = -1$

$x_2 = 5 + 7\lambda$

$x_1 + 3(5 + 7\lambda) + 4(-3 - \frac{11}{2}\lambda) + 5\lambda = 2$

$x_1 = -1 - 4\lambda$

$$\vec{x} = \begin{pmatrix} -1 \\ 5 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 7 \\ -11/2 \\ 1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} -1 \\ 5 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -8 \\ 14 \\ -11 \\ 2 \end{pmatrix}$$