1.1 $\min\left(-4 \mid -1 \mid -8\right)$

1.2 $3x + 6y - z - 8 = 0$

1.3 $\dfrac{82}{13}$

2.1 $t = \dfrac{\sqrt{x^2 + 800^2}}{v} + \dfrac{\sqrt{\left(1000 - x\right)^2 + 200^2}}{2v}$

2.2 426,14

3.1 $x - \frac{1}{6}x^3 = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3\,,\ x_1 = 0{,}5858\,,\ x_2 = 3{,}4142$

3.2 $x_1 = 0{,}5000\,,\ x_2 = 0{,}5856\,,\ x_3 = 0{,}5885\,,\ x_4 = 0{,}5885$

4.1 $P_1\left(0 \mid 0\right),\ P_2\left(-1 \mid 6\right)$

4.2 $\min\left(-1 \mid 6 \mid -4\right)$

4.3 $72x - 12y + z - 68 = 0$

4.4 -8,16

5.1 $1{,}4142 + 0{,}3536\left(x-2\right) - 0{,}0442\left(x-2\right)^2 = 1{,}3863 + \left(x-2\right) - 0{,}2500\left(x-2\right)^2$

$x_1 = 2{,}0438\,, x_2 = 5{,}0973$

5.2 $x_1 = 1{,}6667\,,\ x_2 = 1{,}9981\,,\ x_3 = 2{,}0431\,,\ x_4 = 2{,}0438$

6.1 $P_1\left(0 \mid 0\right),\ P_1\left(1 \mid -1\right)$

6.2 $\min\left(1 \mid -1 \mid -1\right)$

6.3 $x_1 = -4\,,\ x_2 = 2$

6.4 $18x + 30y + z + 20 = 0$

6.5 $F^{\perp} = \dfrac{1}{1225}\begin{pmatrix} 5832 \\ 9720 \\ 324 \end{pmatrix},\ F^{\parallel} = \dfrac{1}{1225}\begin{pmatrix} 6418 \\ -3595 \\ -7674 \end{pmatrix}$

6.6 6

7.1 $1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 = 3 - \frac{3}{2}x^2 + \frac{1}{8}x^4\,,\ x_{1,2} = \pm 1$

7.2 $x_1 = 1{,}0098$

8.1 $P_1\left(-3 \mid -39\right)$, $P_2\left(4 \mid -32\right)$, $\min\left(4 \mid -32 \mid -8192\right)$

8.2 $516x - 444y + z + 168 = 0$

8.3 $F^{\perp} = \begin{pmatrix} 1032 \\ -888 \\ 2 \end{pmatrix}$, $F^{\parallel} = \begin{pmatrix} -999 \\ -1161 \\ 0 \end{pmatrix}$

9 $t: y = -2\xi\left(x - \xi\right) + 4 - \xi^2$

$A\left(\xi\right) = \frac{1}{2}\left(4 + \xi^2\right)\frac{4 + \xi^2}{2\xi}$

$x_{\min} = \frac{2}{3}\sqrt{3}$, $A_{\min} = \frac{32}{9}\sqrt{3}$

10.1 $P_1\left(-\sqrt{2} \mid -1\right)$, $P_2\left(\sqrt{2} \mid -1\right)$

$\min\left(-\sqrt{2} \mid -1 \mid 2\sqrt{2}\right)$, $\max\left(\sqrt{2} \mid -1 \mid -2\sqrt{2}\right)$

10.2 $Q\left(2 \mid -1 \mid -3\right)$

$\vec{n} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\varphi = 23{,}6\,^\circ$

10.3 $v^{\perp} = \begin{pmatrix} 0{,}4 \\ 0 \\ 0{,}8 \end{pmatrix}$, $v^{\parallel} = \begin{pmatrix} -0{,}4 \\ 2 \\ 0{,}2 \end{pmatrix}$

11.1 $\det V = 2 \neq 0$, $\det W = 5 \neq 0$

11.2 $\frac{1}{5}\begin{pmatrix} -8 & -1 & -1 \\ -31 & -12 & -7 \\ 37 & 14 & 9 \end{pmatrix}$

11.3 $\begin{pmatrix} -3 \\ -20 \\ 24 \end{pmatrix}_W$, $\begin{pmatrix} 12 \\ 7 \\ 9 \end{pmatrix}$

11.4 $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\frac{1}{\sqrt{70}}\begin{pmatrix} 6 \\ 5 \\ -3 \end{pmatrix}$, $\frac{1}{\sqrt{14}}\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

12 $f\left(x_0+t\right)+f\left(x_0-t\right)=2y_0\,,\ P_0\left(3\mid -5\right)$

13 $$\lim_{n\to\infty}\frac{a_n}{a_{n+1}}=\lim_{n\to\infty}\frac{a_n}{a_n+a_{n-1}}=\lim_{n\to\infty}\frac{1}{1+\frac{a_{n-1}}{a_n}}=\frac{1}{1+\lim\limits_{n\to\infty}\frac{a_{n-1}}{a_n}}$$

$$\lim_{n\to\infty}\frac{a_n}{a_{n+1}}=\lim_{n\to\infty}\frac{a_{n-1}}{a_n}=x$$

$$x=\frac{1}{1+x}$$

$$x_{1,2}=\frac{-1\pm\sqrt{5}}{2},\ x_2<0\quad\Rightarrow\quad\lim_{x\to\infty}\frac{a_n}{a_{n+1}}=x_1=\frac{\sqrt{5}-1}{2}$$

14 $$f'\left(t\right)=\frac{f\left(b\right)-f\left(a\right)}{b-a}$$

$$f\left(t\right)=\sqrt{1+t}\,,\ a=0\,,\ b=x\,,$$

$$f'\left(t\right)=\frac{1}{2\sqrt{1+t}}=\frac{\sqrt{1+x}-\sqrt{1+0}}{x-0}=\frac{\sqrt{1+x}-1}{x}$$

$$\frac{1}{2\sqrt{1+t}}=\frac{\sqrt{1+x}-1}{x}$$

$$\frac{x}{2}=\sqrt{1+t}\cdot\left(\sqrt{1+x}-1\right)$$

$$t\in\left]0;x\right[\quad\Rightarrow\quad\sqrt{1+t}<\sqrt{1+x}\quad\Rightarrow\quad\frac{x}{2}<\sqrt{1+x}\cdot\left(\sqrt{1+x}-1\right)$$

$$\frac{x}{2}<1+x-\sqrt{1+x}$$

$$\sqrt{1+x}<1+\frac{1}{2}x$$

15.1 $n=1:\quad 1=\dfrac{1\cdot 2\cdot 3}{6}\quad\checkmark$

$$\sum_{k=1}^{n+1}k^2=\frac{n\left(n+1\right)\left(2n+1\right)}{6}+\left(n+1\right)^2=\frac{n\left(n+1\right)\left(2n+1\right)+6\left(n+1\right)^2}{6}$$

$$=\frac{\left(n+1\right)\left(n\left(2n+1\right)+6\left(n+1\right)\right)}{6}=\frac{\left(n+1\right)\left(2n^2+7n+6\right)}{6}$$

$$=\frac{\left(n+1\right)\left(n+2\right)\left(2n+3\right)}{6}=\frac{\left(n+1\right)\left(n+1+1\right)\left(2\left(n+1\right)+1\right)}{6}\quad\checkmark$$

15.2 $n=1:\quad 1=1\quad\checkmark$

$$\sum_{k=1}^{n+1}\left(2k-1\right)=n^2+\left(2\left(n+1\right)-1\right)=n^2+2n+1=\left(n+1\right)^2\quad\checkmark$$

16.1 $x_0 = -1\,,\ r = \lim_{n\to\infty} \dfrac{3n}{3\left(n+1\right)} = 1$

$x = -1 - 1 = -2: \quad \sum_{n=0}^{\infty} 3n\left(-1\right)^n$ divergiert

$x = -1 + 1 = 0: \quad \sum_{n=0}^{\infty} 3n\left(1\right)^n$ divergiert

Konvergenz in $\left]-2;0\right[$

16.2 $\sum_{n=1}^{\infty} \dfrac{\left(4x-1\right)^n}{n^n} = \sum_{n=1}^{\infty} \dfrac{4^n}{n^n}\left(x - \dfrac{1}{4}\right)^n$

$x_0 = \dfrac{1}{4}\,,\ r = \dfrac{1}{\overline{\lim\limits_{n\to\infty}}\sqrt[n]{\dfrac{4^n}{n^n}}} = \dfrac{1}{\overline{\lim\limits_{n\to\infty}}\dfrac{4}{n}} = \infty$

Konvergenz in $\mathbb{R}$ (bzw. $\mathbb{C}$).